

Letter of Sasha Borovik, 24 November 2018  
on problem of 101 coins

So everything boils down to a statement about natural numbers:

*We are given 101 natural numbers, and after removal of any one of them from the list the remaining numbers can be redistributed in two groups of 50 numbers each in such a way that the sum of numbers in each group is the same. Prove that all numbers are equal.*

*Here is a re-write of the solution suitable for junior school kids.*

After removal of any of numbers the sum of the remaining numbers is even. Hence all numbers have the same parity: either they are all even, or they are all odd. In the first case, divide all numbers by 2. In the second case, subtract 1 from all numbers. In both cases we get the same problem, but with smaller numbers, and this continues until one of the numbers is 0. If numbers are all zero, we are done. Assume that there is a non-zero number left. Notice that we can no longer make further subtractions of 1, but only divisions by 2, and we continue them until some other number becomes odd. At this point, either removal of 0, or removal of that odd number creates 100 numbers with the odd sum, a contradiction.

*Ideologically this is a hidden calculation in base 2 arithmetic (this is why 2-adic numbers appeared in Karabegov's arithmetic (this is why 2-adic numbers appeared in Karabegov's beautiful solution), alike the Russian Peasants' Multiplication or Russian Peasants' Greatest Common Divisor. Pedagogically, this is a nice example of the use of abstraction: in the original formulation with coins, the problem is next to impossible for a schoolchild – you cannot subtract 1 from the value of a coin, it is called debasement of the coinage and, I am afraid, is likely to remain a criminal offence in this country. It helps to show to a child that he/she can forget about coins and get freedom of action – the child will be likely to look for this freedom in other problems as well. Historically, in my previous incarnation I had a chance to see, and participate in, creation of mathematical olympiad problems – this was the same kind of process as the one demonstrated to us by Hovik. Thanks to Hovik, I had a chance to revive sweet memories of my younger years. Alas, nostalgia is not what it used to be ...*

Best wishes – Sasha