p-adic numbers and Peano curve

I learned few days ago how p-adic numbers help to construct Peano curve. We use ring of 2-adic integers $\mathbf{Z}_2$ in intermediate step. In the final construction p-adic numbers dissapear. This is a good example of their real value (the construction belongs to Shindiapin)

Let I be closed segment $[0,1]$ and C be Cantor set in I:

$$C = \left\{ x \in I\colon\ x = \sum_{k=1}^{\infty} \frac{a_k}{3^k}, \quad a_k = 0, 2 \right\}$$

Consider the following map from Cantor set C to the square $I \times I$ defined as composition of the maps:

$$C \xrightarrow{f} C \times C \xrightarrow{F\times F} \mathbf{Z}_2 \times \mathbf{Z}_2 \xrightarrow{G\times G} I \times I \tag{1}$$

Here

1) Map f is a map defined as

$$f\left(\sum_{k=1}^{\infty} \frac{a_k}{3^k}\right) = \left(\sum_{k=1}^{\infty} \frac{a_{2k}}{3^k}, \sum_{k=1}^{\infty} \frac{a_{2k-1}}{3^k}\right) \tag{3}$$

i.e. $f(0.a_1a_2a_3a_4) = (0.a_2a_4a_6...;0,\ a_1a_3a_5...)$ (all a_i are 0 or 2).

2) Map $F\colon\ C \to \mathbf{Z}_2$ is defined as

$$F\left(\sum_{k=1}^{\infty} \frac{a_k}{3^k}\right) = \sum_{k=0}^{\infty} \frac{a_{k+1}}{2} 2^k, \qquad \text{all } a_i \text{ are 0 or 2.} \tag{4}$$

and map G is defined as

$$G\left(\sum_{k=0}^{\infty} a_k 2^k\right) = \sum_{k=0}^{\infty} \frac{a_k}{2^k} \tag{5}$$

One can prove that all these maps are continuous maps, the map (4) is homeomorphism, and the map (5) is surjection. Hence the composition (1) is continuous surjection of the Cantor set C on the square $I \times I$. One can expand this map by linearity on the all set I.

Thus we come to continuous map from I on $I \times I$. This map is continuous in usual topology on real numbers. We come to Peano curve.

Mavr sdelal svojo delo, mavr mozhet umeretj!