## Geometrical object (look from outside)

*Long ago I red the definiiton of geometrical object in Beklemishev (Manuel on Analytical geometry). I was very happy to know how physisists define vectors, tensors,... and honestly long time did not understand the question: "does it exist"...., and the fact that it is the cocycle condition (see equation (3) below which guarantees it!))*

*I struggled with this during all my life. My friend Ted Voronov always emphasized the importance of cocycle condition for the existence of object.. The proverb: "vek zhivi, vek uchisj, durakom pomrjoshj" works fine in this case.*

**Definition** We say that at the points of manifold $M$ we define `geometrical object` if for every point $\mathbf{pt} \in M$ and for every local coordinates $x^i_{(\alpha)}$ which are defined in the vicinity of this points we define the sequence $\Sigma^I$ of numbers

$$\{x^i_{(\alpha)}\} \mapsto \quad \Sigma^I = \Sigma^I\left(x^i_{(\alpha)}\right) \tag{1}$$

such that the following conditions are obeyed:

1) Let $x^i_{(\alpha)}$ and $x^{i'}_{(\beta)}$ be two arbitrary coordinates which are defined in the vicinity of the point $\mathbf{pt}$, then

$$\Sigma^{I'}\left(x^{i'}_{(\beta)}\right) = \Psi^{I'}_{\beta\alpha}\left(\Sigma^I\left(x^i_{(\alpha)}\right)\right)\,. \tag{2}$$

2) Let $x^i_{(\alpha)}$, $x^{i'}_{(\beta)}$, $x^{i''}_{(\gamma)}$ be three arbitrary coordinates which are defined in the vicinity of the point $\mathbf{pt}$, and let object $\Sigma$ is defined

$$\begin{array}{c}
\text{in coordinates } \{x^i_{(\alpha)}\} \text{ by the sequence } \Sigma^I_\alpha\left(x^i_{(\alpha)}\right) \\
\text{in coordinates } \{x^{i'}_{(\beta)}\} \text{ by the sequence } \Sigma^{I'}_\beta\left(x^{i'}_{(\beta)}\right) \\
\text{in coordinates } \{x^{i''}_{(\gamma)}\} \text{ by the sequence } \Sigma^{I''}_\gamma\left(x^{i''}_{(\gamma)}\right)
\end{array}$$

Then

$$\Psi^I_{\alpha\gamma}\left(\Sigma^{I''}_\gamma\left(x^{i''}_{(\gamma)}\right)\right) = \Psi^I_{\alpha\beta}\left(\Psi^{I'}_{\beta\gamma}\left(\Sigma^{I''}\left(x^{i''}_{(\gamma)}\right)\right)\right) = \tag{3}$$

Equation (2) defines transformation of components of geometrical object when we transform coordinates.

Equations (3) (cocycle conditions ) guarantee that if you transform coordinates in different ways, the answer is idependent. For example if we have five different local coordinates

$$x_{(\alpha)}\,, x_{(\beta)}\,, x_{(\gamma)}\,, x_{(\delta)}\,, x_{(\rho)}\,, x_{(\omega)}\,,$$

such that

$$x_{(\alpha)}\left(x_{(\beta)}\left(x_{(\gamma)}\left(x_{(\delta)}\right)\right)\right) = x_{(\alpha)}\left(x_{(\rho)}\left(x_{(\omega)}\left(x_{(\delta)}\right)\right)\right)$$

then the coycle condition (3) guqrantees that answer will be the same if you go from coordinates $x_{(\delta)}$ to coordinates $x_{(\alpha)}$ through coordinates $x_{(\gamma)}$ and $x_{(\beta)}$ or if you go from coordinates $x_{(\delta)}$ to coordinates $x_{(\alpha)}$ through coordinates $x_{(\omega)}$ and $x_{(\rho)}$ or if you go.

Cocycle conditions guarantees existence of the object.

Examples? It is crazy easy. Remember how physisists define vectors. The define vector as array of components which transforms in a given way if you change a basis. It is only sometimes very curious students ask question: does at least one vector exist *?

* the existence is guaranteed by the condition...[try to write it!!!] (Usually this question makes angry a lecturer if his (her) level of mathematical culture is not too high)