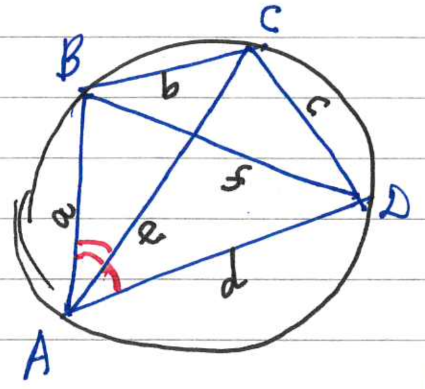
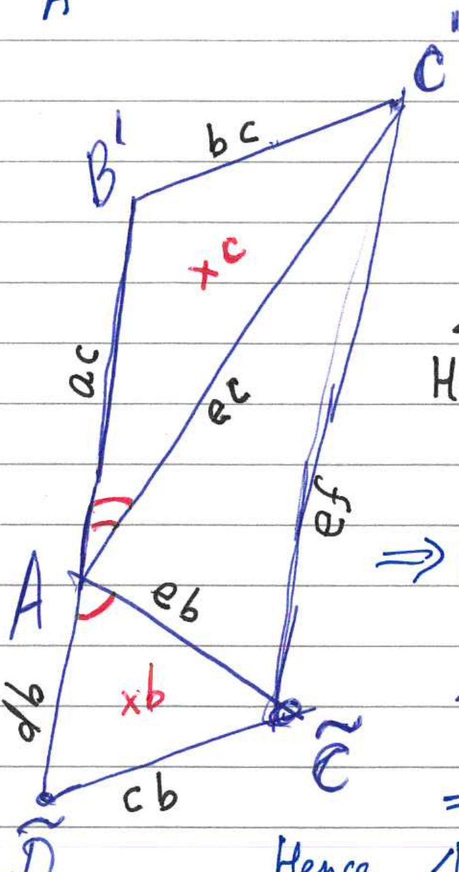
Ptolemy's Theorem



1. Rotate triangle ΔACD around vertex A clock-wise, such that D, A, B become on one line

2. Multiply ΔABC on c and ΔACD on b (homothety centre is vertex A).



$\Delta AB'C' \sim \Delta ABC$, $\Delta A\tilde{C}\tilde{D} \sim \Delta ACD$.

$$\angle A\tilde{D}\tilde{C} + \angle AB'C' = \angle ADC + \angle ABC = \pi$$

Hence $B'C' \parallel \tilde{D}\tilde{C}$, $B'C' = \tilde{D}\tilde{C} = bc$. $\Longrightarrow$

$\Rightarrow B'C'\tilde{C}\tilde{D}$ is parallelogram

$$\angle \tilde{C}AC' = \pi - \angle \tilde{C}A\tilde{D} - \angle C'AB' =$$

$$= \pi - \angle CAD - \angle BAC = \angle BCD.$$

Hence $\Delta C'A\tilde{C} \sim \Delta CDA$

$(C'A = ec,\ \tilde{C}A = eb,\ \angle C'A\tilde{C} = \angle DCB)$

Hence $C'\tilde{C} = ef$

In parallelogram $\tilde{D}B = C'\tilde{C} \Longrightarrow ac + db = ef$

This proof I heard from Anna Felikson.

Thank you, Anna

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