Homework 8

1. Find unit normal vector field, the Weingarten (shape) operator, principal curvatures and the Gaussian curvature for the cylinder $x^2 + y^2 = a^2$.

1. Find unit normal vector field, the Weingarten (shape) operator, principal curvatures and the Gaussian curvature for the sphere of the radius R: $x^2 + y^2 + z^2 = R^2$.

3 Find the Weingarten (shape) operator the Gaussian curvature for the saddle $z = xy$ at the point $x = y = 0$.

4 Let D be a domain on a sphere of radius R such that its area is equal to one eighth of the area of the sphere. Calculate the integral of Gaussian curvature of the sphere over the domain D.

What is a result of parallel transport of the vector, tangent to the sphere, along the curve $C = \partial D$?

5 Assume that the action of the shape operator at the tangent coordinate vectors $\mathbf{r}_u = \partial_u$, $\mathbf{r}_v = \partial_v$ at the given point $\mathbf{p}$ of the surface $\mathbf{r} = \mathbf{r}(u, v)$ is defined by the relations: $S(\partial_u) = 2\partial_u + 2\partial_v$ and $S(\partial_v) = -\partial_u + 5\partial_v$. Calculate principal curvatures and Gaussian of the surface at this point.

6 Consider a surface M, the upper sheet of the cone in $\mathbf{E}^3$

$$\mathbf{r}(h, \varphi): \quad \begin{cases} x = 3h\cos\varphi \\ y = 3h\sin\varphi \\ z = 4h \end{cases}, \quad h > 0\,,\ 0 \le \varphi < 2\pi\,.$$

Using properties of Weingarten operator show that Gaussian curvature vanishes at all the points of this surface.

Let C_1 be a closed curve on this surface which is the boundary of a compact oriented domain $D \subset M$.

Let C_2 be a circle which is the intersection of the plane $z = h_0$ ($h_0 > 0$) with the surface M.

Show that the parallel transport along the closed curve C_1 is the identical transformation.

Show that the parallel transport along the closed curve C_2 is the rotation through a non-zero angle.

Calculate this angle and explain why the fact that the angle is not equal to zero does not contadict to the Theorem on parallel transport.