Homework 5.

1 Consider function $f = xy$ and differential forms $\sigma = xdy + ydx$ and $\omega = zdx + xdy$. Calculate differential forms $d(f\omega)$, $\sigma \wedge \omega$ and $d(\sigma \wedge \omega)$.

2 Consider embedding $\iota\colon S^2 \to \mathbf{R}^3$ of the sphere to $\mathbf{R}^3$ given by the equation $x = a\sin\theta\cos\varphi$, $y = a\sin\theta\sin\varphi$, $z = a\cos\theta$. Calculate pull-backs
$\iota^*(f)$,
$\iota^*(\sigma)$,
$\iota^*(\omega)$,
$\iota^*(\sigma \wedge \omega)$,
$\iota^*\left(d(\sigma \wedge \omega)\right)$.

3 Consider the embedding $\iota\colon M \to \mathbf{R}^2$ of the circle S^1 in $\mathbf{R}^2$ given by the equation $x = a\cos\theta, y = a\sin\theta$. Find the pull-backs $\iota^*(\sigma)$ and $\iota^*(d\sigma)$ if $\sigma = \frac{xdy - ydx}{x^2+y^2}$.

4 Consider the embedding $\iota\colon M \to \mathbf{R}^3$ of the cylinder M in $\mathbf{R}^3$ given by the equation $x = a\cos\theta, y = a\sin\theta, z = h$. Find the pull-backs $\iota^*(\sigma)$ and $\iota^*(d\sigma)$ for the following forms:
$\sigma = zdy$
$\sigma = xdy + ydx$
$\sigma = \frac{xdy - ydx}{x^2+y^2}$,
$\sigma = xdy - ydx$

5* Show that $\iota^*(\sigma) = \sin\theta d\theta \wedge d\varphi$, where ι is embedding of the sphere in $\mathbf{R}^3$ considered in the exercise 2, and the 2-form σ is defined by the formula

$$\sigma = \frac{xdy \wedge dz + ydz \wedge dx + zdx \wedge dy}{(x^2+y^2+z^2)^{3/2}}$$

6 Calculate differential of 1-form $\alpha = p_1 dq^1 + p_2 dq^2 - dp_2 q^2$

7 Consider in $\mathbf{R}^2$ a triangle $\triangle ABC$ with vertices at the points $A = (5, -1)$, $B = (-1, 6)$, $C = (-5, -1)$ and differential one-form $\omega = xdy - ydx$. By using Stokes' theorem or directly calculate the integral of 1-form ω over the boundary of the $\triangle ABC$.

8 Consider in $\mathbf{R}^3$ the surface defined by the equation $x^2 + y^2 + z^2 = 4z$. Show that this surface is the sphere.

Using Stokes Theorem calculate the integrals of the 2-forms $\omega_1 = zdx \wedge dy$ and $\omega_2 = dx \wedge dy$