Homework 6.

1 Calculate $H^0_{DR}(M)$ in the case if manifold M is

a) $M = \mathbf{R}$,

b) $M = S^1$,

c) M=$\mathbf{R}^n$,

d) $M = \mathbf{R}^n \backslash \{0\}$ $(n = 1, 2, 3, \dots)$,

e) M is an arbitrary topological manifold.

2 Calculate all de Rham cohomology groups $H^k_{DR}(M)$ for

a) $M = \mathbf{R}^1$,

b) $M = S^1$.

Compare answers.

3 Calculate all de Rham cohomology groups $H^k_{DR}(M)$ for manifolds

a) $N = S^1$,

b) $M = \mathbf{R}^2 \backslash \{0\}$.

Compare answers

4 Consider the space $M = \mathbf{R}^2 \backslash \{0\}$. Show that it is not a star-shaped domain.

Consider the form

$$\omega = \frac{xdy - ydx}{x^2 + y^2} \,. \tag{1}$$

Calculate the integral of this 1-form over

a) a unit circle S^1 $(x^2 + y^2 = 1)$,

b) over closed curve which is a boundary of domain which contains an origin,

c) over a closed curve which is a boundary of domain which does not contain an origin.

5* Let σ be an arbitrary closed 1-form on $M = \mathbf{R}^2 \backslash \{0\}$ and an integral of this form over the unit circle $x^2 + y^2 = 1$ is equal to q. Consider the form $\sigma' = \sigma - \frac{q}{2\pi}\omega$, where ω is the form defined by the equation (1). Show that this an exact form

Hint (solution????): One can see that an integral of this form over the unit circle $x^2 + y^2 = 1$ is equal to 0, hence consider the scalar function $\Phi(x)$ which is equal to the integral of σ' over the curve finishing at the point x. One can see that $d\Phi = \sigma'$

6 a) Calculate de Rham cohomology groups for $\mathbf{R}^2$.

b) Show an example of closed two form non-homologous to zero on S^2. Calculate de Rham cohomology groups for S^2. Explain why $H^2_{DR}(S^2 \backslash N) = 0$.

7 Calculate de Rham cohomology groups for $\mathbf{R}^3$. What can be said about de Rham cohomology groups of the space $\mathbf{R}^3 \backslash 0$ in comparison with $\mathbf{R}^3$ and S^2?