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Secondary Quantisation

In previous lecture considering hermonic oscillator we used expressions like 'creation' operator, 'annihilation' operator. . . .

To day we establish isomorphism between

Non-relativistic Quant. Mech. of MANY non-interacting particles $\Longleftrightarrow$ Many non-interacted hermonic oscyllators

$$\sum_i \hbar\omega_i\left(\hat{a}_i^+ a_i + \frac{1}{2}\right)$$

Let $H = \frac{p^2}{2m} + U(q)$ be Hamilt. of particle

classic

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(q) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + U(q)$$

$$\hat{H}\varphi_n = E_n\varphi_n \Rightarrow \Psi(x,t) = \sum C_n(t)\varphi_n(x),$$

$$\Uparrow$$

$$\boxed{\frac{dC_n}{dt} = i\hbar E_n C_n} \quad i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

$(C_n = p_n + i q_n)$

Diff. eq. describing one QUANTUM particle

$$\|$$

Hamiltonian equation:

$$\mathcal{H} = \sum E_n \bar{C}_n C_n, \qquad \{C_n, \bar{C}_m\} = i\hbar\,\delta_{nm}$$

$(C_n, \bar{C}_n \in \mathbb{C})$

$$\dot{C}_n = \{C_n, H\} = i\hbar E_n C_n.$$

$$H = \frac{p^2}{2m} + U(q)$$

$$\downarrow$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(q) \Longleftrightarrow \mathcal{H} = \sum E_n \bar{c}_n c_n, \ \{\ ,\ \}$$

Quantisation of $\mathcal{H}$

$$\hat{\mathcal{H}} = \sum E_n \hat{a}_n^+ a_n$$

||

describes many non-interacting particles

n_1 particles on the level E_1

n_2 " ——— " E_2

n_k " ——— " E_k

$\hat{a}_i^+$ increases on one number of particles ~~on the~~ with Energy E_i

$$\hat{\mathcal{H}} = \sum E_n \left(\hat{a}_n^+ a_n + \frac{1}{2}\right)$$

k-th oscillator has Energy $\hbar\omega_k(n_k + \frac{1}{2})$

$$\hat{a}_i^+ |n_1 n_2 \dots n_i \dots\dots\rangle =$$

$$= \sqrt{n_i + 1}\, |n_1 n_2 \dots n_i + 1 \dots\rangle$$

$\hat{a}_i^+$ increases level of energy of i-th oscyllator