Lecture VIII

Perturbations in Quant. Mech. ⟹ Adiab. inv.

1) Stationary case.

$$\hat{H} = \hat{H}_0 + \varepsilon \hat{V}$$

$$\hat{H}\varphi_n = E_n \varphi_n$$

$$\varphi_n = \varphi_n^{(0)} + \varepsilon \varphi_n^{(1)} + \varepsilon^2 \varphi_n^{(2)} + \ldots$$

$$E_n = E_n^{(0)} + \varepsilon E_n^{(1)} + \varepsilon^2 E_n^{(2)} + \ldots$$

$$E_n = E_n^{(0)} + \varepsilon V_{nn} + \varepsilon^2 \sum_{m \neq n} \frac{V_{nm}^* V_{mn}}{E_m - E_n}$$

virtual transition

psychological Effect

~~through~~

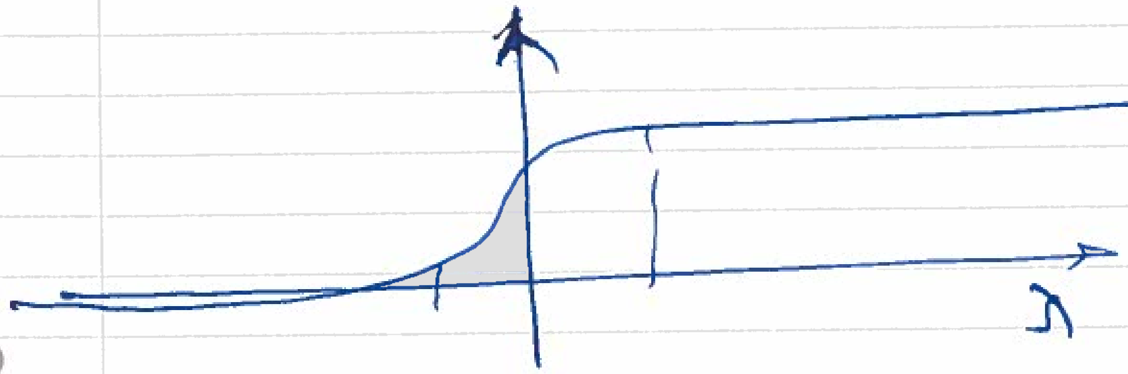
~~tr~~

trough

I newer will go to ~~see~~ this island,
but why you
do not allow me
to do it,

Adiabatic and abrupt perturbation theory:

$$H(\lambda) = \hat{H}_0 + \hat{V}(\lambda)$$



$\hat{H}_0 - \hat{V}(\lambda) = 0$ if $\lambda < -1$, $\quad \hat{H}(\lambda) = \hat{H}_0$.

$\hat{V}(\lambda) = V_1$ if $\lambda > 1$ $\quad \hat{H}(\lambda) = \hat{H}_1$

Hamiltonian changes

$$H_0 \longrightarrow H_1.$$

1) $\lambda = \varepsilon t$ $\quad \varepsilon \to 0$ $\quad -\frac{1}{\varepsilon} < t < \frac{1}{\varepsilon}$

adiabatic

2) $\lambda = \varepsilon t$ $\quad \varepsilon \to \infty$ $\quad$ Abrupt

$\lambda = \varepsilon t$

$$\begin{cases} i\hbar \dfrac{\partial \Psi}{\partial t} = \hat{H}(\lambda)\Psi \\ \Psi \underset{t \to -\infty}{=} \varphi_n(x) e^{-\frac{iE_n t}{\hbar}} \end{cases}$$

$$\hat{H}(\lambda)\varphi_n(\lambda) = E_n(\lambda)\varphi_n(\lambda)$$

$$\Psi(x,t) = \exp\left[-\frac{i}{\hbar}\int^{T} E(t)\,dt\right]\left[\varphi_n(\varepsilon t) + \dots\right]$$

Hierarchy of function is not changed.

1-st function remains 1-st

2-nd " ——————— " 2-nd

n-th " ——————— " n.

Number (numero) is preserved.

soft inflation does not change ranking

Abrupt

$\hat{H}(\lambda)$ $\lambda = \varepsilon t$ $\varepsilon \to \infty$

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}(\varepsilon t)\Psi$$

$$\underset{t<\infty}{\Psi(t)} = e^{-\frac{iE_n^{(0)}t}{\hbar}}\varphi_n^{(0)} \quad [H_0\varphi_n^{(0)} = E_n^{(0)}\varphi_n^{(0)}]$$

$\Psi(t)$ is not changed, but it ceased to be stationary function.

$\underset{t<0}{\Psi(t) = e^{-\frac{iEt}{\hbar}}\varphi_1}$ φ_1 is not stationary

Ψ' You go to bed in one country and get up in another.

Adiabatic perturb.

Quantum Mech. —— Classical m.

n — remains unchanged

$$\boxed{2\pi\hbar n \sim \int p\,dq}$$

Born-Sommerfeld

n - is preserved

$$\int_{\text{along traj}} p\,dq = \int_S dp \wedge dq$$

$$\|$$

Adiab. invariant in classical Mechanics

$$H(\lambda) = \frac{p^2}{2m} + U(q,\lambda)$$

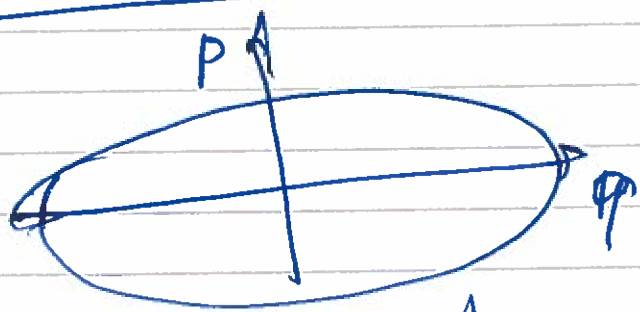
$I(p,q,\lambda)$ – adiabatic inv.

$$I(p(t), q(t), \lambda = \varepsilon t) =$$

$$= I(p(0), q(0), \lambda = 0) + O(\lambda = \varepsilon t)$$

$$\forall \varkappa \ \exists \varepsilon : |O(\varepsilon t)| < \varkappa \text{ if } 0 < t < \frac{1}{\varepsilon}$$

Fix λ



$$I = S = 2\int p\,dq = \sqrt{2m}\int_{q_1}^{q_2}\sqrt{H-U}$$

Example.

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} \qquad \omega = \omega(\lambda)$$

$$\boxed{S = \frac{H(p,q,\lambda)}{\omega(\lambda)}}$$