Quantum mechanics. Problems 4.

Heisenberg uncertainty principle

One of the formulation of the Heisenberg incertainty principle is the following: Let $\hat{A}, \hat{B}$ be two observables, and $\hat{C}$ be an operator such that

$$\hat{C} = i[\hat{A}, \hat{B}] \,. \tag{1}$$

Then for any given state $\Psi \neq 0$ the product of dispersions of observables $\hat{A}$ and $\hat{B}$ is bigger tor equal than the half of the dispesion of $\hat{C}$.*

1. a) is the operator $C' = -iC = [A, B]$ in (1) an observable?
b) explain why the operator $C = i[A, B]$ is an observable.

2 Suppose that for the state $\Psi \neq 0$,

$$< A >_{\text{average}} = a \,, \quad < B >_{\text{average}} = b \,.$$

Show that averages of operators $A' = A - a$ and $B' = B - b$ on the state Ψ vanish.

3 Show that the dispersions of operator A and operator B on the state Ψ coincide respectively with the dispersions of the operator A' and operator B' on this state

4 Using this result prove the Heisenberg uncertainty principle.

5 (`Uncertainty principle in music`) Consider the signal

$$A(t) = \begin{cases} 0 \text{ for t} < 0 \\ A_0 \sin w_0 t \text{ for } 0 < t < T \\ 0 \text{ for} t > T \end{cases}$$

Calculate the dispersion $\sqrt{\Delta w^2}$ of frequency.
Explain why $\sqrt{\Delta w^2} \to \infty$ if $T \to 0$.

6

Consider a state

$$\Psi_1(x, y, z) = e^{(x - x_0)^2 + i\frac{p_0 y}{\hbar}}$$

Is it possible to measure simultaneously x and p_y for this state?

Does this contradicts to Heisenberg uncertainty principle?

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* *Dispersion of operator $\hat{K}$ on an arbitrary state Φ is equal to*

$$\sqrt{\Delta K^2} = \sqrt{< \Phi, (\hat{K} - k)\Phi >} \,,$$

where $k =< \Psi, \hat{K}\Psi >$ is the average of the operator $\hat{K}$. (We suppose that $< \Psi, \Psi >= 1$.)