Quantum mechanics. Problems 9.
Perturbation theory

1 Deduce the formula

$$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \ldots = E_n^{(0)} + V_{nn} + \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^{(0)} - E_k^{(0)}}$$

2 Calculate energy levels for free 1-dimensional Hamiltonian, which is perturbed by δ-function like Hamiltonian:

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \alpha\delta(x - x_0) = -\frac{\hbar^2}{2m}\partial_x^2 + \alpha\delta(x - x_0)\,.$$

Compare the exact answer with the answer obtained by perturbation theory.

3 `Discussion of abrupt and adiabatic perturbation theory`