5 December

Lecture VIII

Stationary perturbation theory

$$\hat{H} = \hat{H}_0 + \varepsilon \hat{V}$$

$$\hat{H} \varphi_n = (\hat{H}_0 + \varepsilon \hat{V}) \varphi_n = E_n \varphi_n$$

$$\varphi_n = \varphi_n^{(0)} + \varepsilon \varphi_n^{(1)} + \varepsilon^2 \varphi_n^{(2)} + \dots$$

$$E_n = E_n^{(0)} + \varepsilon E_n^{(1)} + \varepsilon^2 E_n^{(2)} + \dots$$

$$(\hat{H}_0 + \varepsilon \hat{V})(\varphi_n^{(0)} + \varepsilon \varphi_n^{(1)} + \dots) =$$

$$= (E_n^{(0)} + \varepsilon E_n^{(1)} + \dots)(\varphi_n^{(0)} + \varepsilon \varphi_n^{(1)} + \dots)$$

$$\langle \varphi_n^{(0)}, \varphi_m^{(0)} \rangle = \delta_{nm},$$

Do it ~~in~~ in orders by ε

0) $\varepsilon = 0$

$$\hat{H}_0 \varphi_n^{(0)} = E_n^{(0)} \varphi_n^{(0)}$$

1) ε: $\varepsilon^2 = 0$

$$\varphi_k^{(0)}, \quad \hat{H}_0 \varphi_n^{(1)} + \hat{V} \varphi_n^{(0)} = E_n^{(0)} \varphi_n^{(1)} + E_n^{(1)} \varphi_n^{(0)}$$

~~Choose $k=n$~~

$$\boxed{\langle \varphi_k^{(0)}, \hat{H}_0 \varphi_n^{(1)} \rangle + \langle \varphi_k^{(0)}, \hat{V} \varphi_n^{(0)} \rangle = E_n^{(0)} \langle \varphi_k^{(0)}, \varphi_n^{(1)} \rangle + E_n^{(1)} \delta_{kn}}$$

Choose $k=n$

$$\Downarrow$$

$$\boxed{E_n^{(1)} = V_{nn}}$$

Continuing in the same way we will come to.

$$E_n^{(2)} = \sum_{m \neq n} \frac{V_{nm}^* V_{mn}}{E_m - E_n}$$

$$E_n = E_n^{(0)} + \varepsilon V_{nn} + \sum_{m \neq n} \frac{V_{nm}^* V_{mn}}{E_n^{(0)} - E_n^{(0)}}$$

virtual transition

psycological effects.

I never will go to this island, but why you do not allow me to do this.

What happens if level is degenerate?

$$\hat{H}_0 \varphi_i = E_0 \varphi_i$$

$$i = 1, \dots, N$$

$$\hat{H}_0 \longrightarrow \hat{H}_0 + V$$

$$(\hat{H}_0 + \varepsilon V)\Phi_i = \left(E_0 + \varepsilon E_1^{(i)}\right)\Phi_i$$

$$\Phi_i = \sum C_{ik}^{(i)} \varphi_k$$

$$\Downarrow$$

$$V_{km} C_m^{(i)} = \cancel{\varepsilon} E_1^{(1)} C_k$$

$$\Downarrow$$

$$\det\left(V_{km} - E_1^{(i)}\right) = 0$$

(secular equation)

- $$\frac{\hat{p}^2}{2m} + U(r)$$

$$\hat{H}\left(\hat{O}\,\Psi(\vec{r})\right) = \hat{O}\left(\hat{H}\,\Psi(r)\right)$$

- $$V_E = \{ \Psi : \hat{H}\Psi = E\Psi \}$$

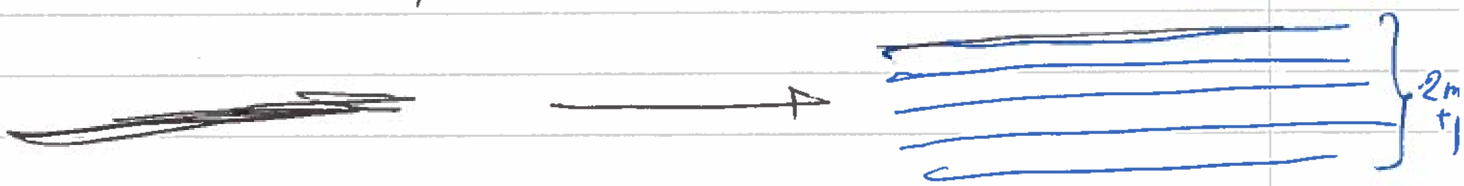
V_E - representation space of $SO(3)$

$$\dim V_E = 2m + 1$$

$$1,\ 3,\ 5,\ 7,\ \ldots$$

- If we put atom in magnetic field

$SO(3)$-symmetry $\longrightarrow$ $SO(2)$-symm.



Zeeman effect

atom in magnetic field

$SO(3)$ - invariant

+

magnetic

$$H_0 \varphi_i = E_0 \varphi_i$$

$i = 1, \ldots, N$

$N =$ dim of irr. represent of $SO(3)$

$N = 2m+1$ $\longrightarrow$ $SO(2)$

The level is splitted on $2m+1$ levels

$$\dim V = 2m+1^{*}$$

* There is degeneracy related with hidden symmetry of hydrogen atom (Runge - Lentz vector).

Time - dependent perturbation.

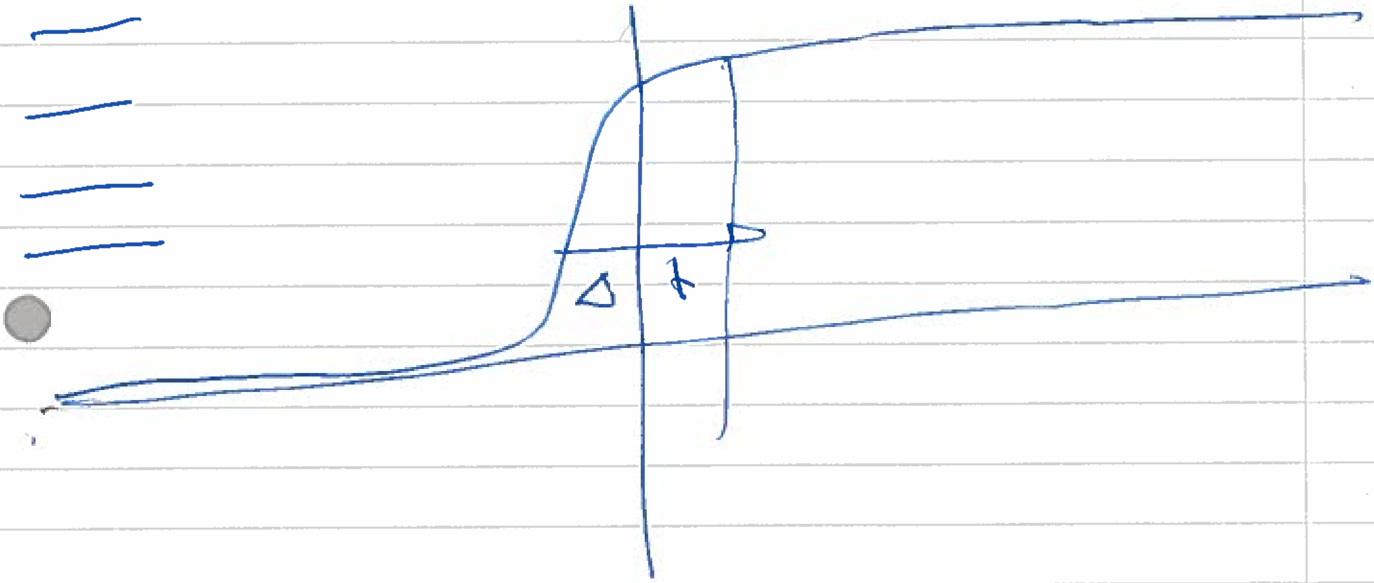
$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}(t)\,\Psi$$

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\Psi(t) = \sum C_n(t)\,\varphi_n(x) \quad \left(\hat{H}_0 \varphi_n = E_n \varphi_n\right)$$

$$i\hbar \frac{d C_n(t)}{dt} = V_{nm}\, C_m.$$

$$C_n(t) = \frac{1}{i\hbar} \int_0^{n} V_{nm}(\tau)\, C_m(\tau)\, d\tau$$



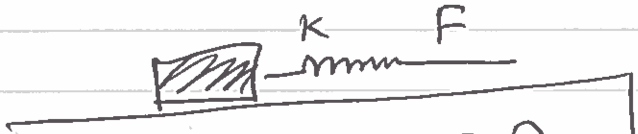
$$\Delta t \sim \frac{1}{\Delta \omega}$$

Example:

Abrupt perturbation

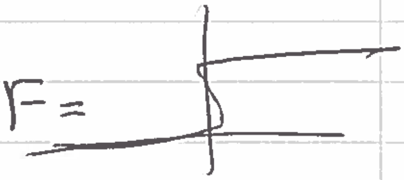
$$\Delta t \ll \Delta \omega$$

Example



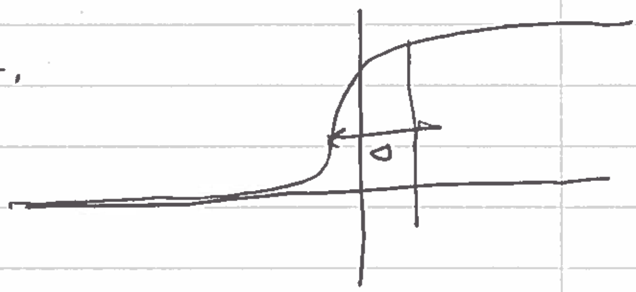
$F = \frac{\mu m g}{2}$??

$F = \mu m g$!!

$F =$ 

Two char. times.

Δt_1 = time of force est.



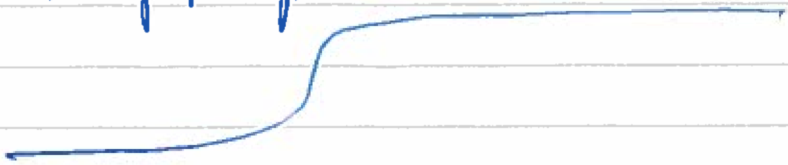
$$\Delta t_2 = \frac{1}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

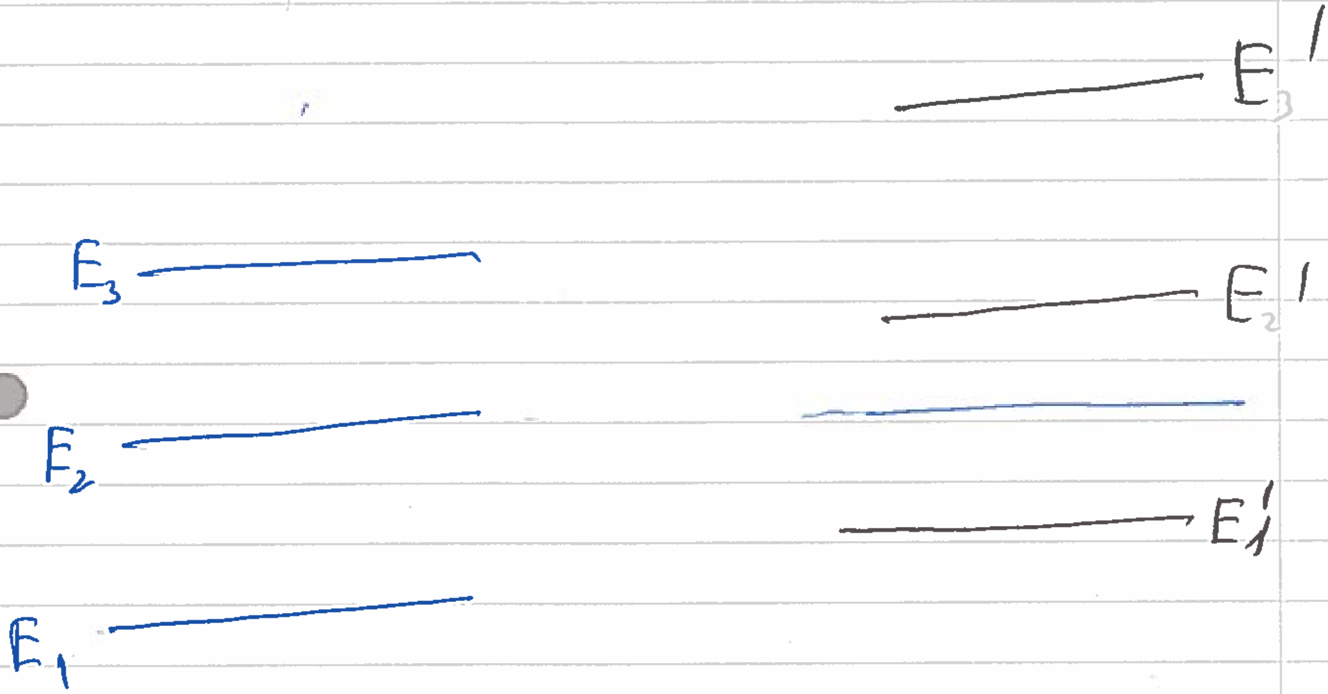
$$\lim_{\Delta t_2 \to 0} \lim_{\Delta t_1 \to 0} F = \frac{\mu m g}{2}$$

$$\lim_{\Delta t_1 \to 0} \lim_{\Delta t_2 \to 0} = \mu m g$$

Phase transition!

Abrupt perturbation





Wave-function DOES NOT change

$\Psi = \varphi_2$ – stationary

$\varphi_2 = a\psi_1' + b\psi_2'$

becomes oscillating!

$\boxed{\text{You go to bed in one country and get up at the morning in another}}$