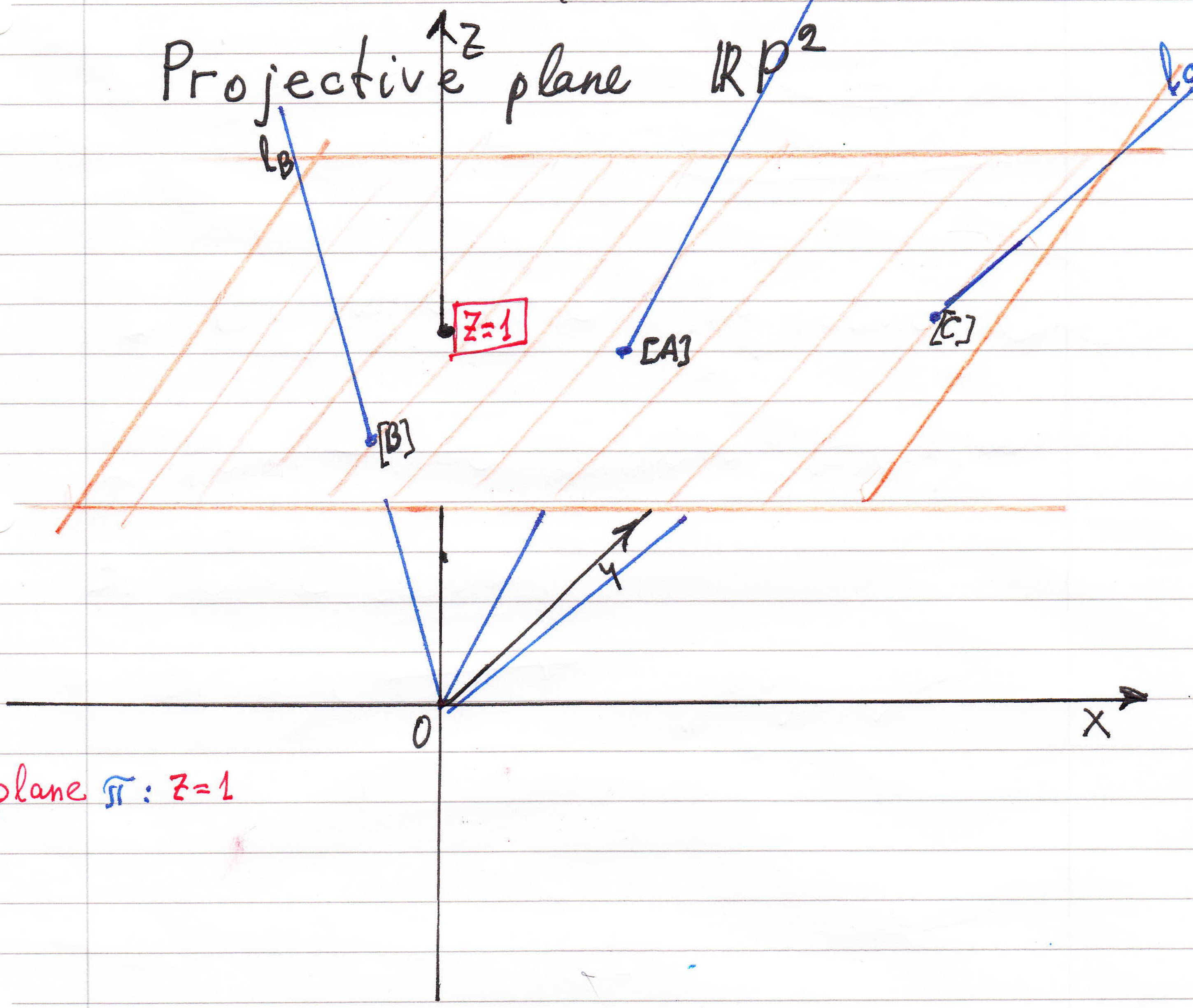
# Lecture CIX

## Projective plane $\mathbb{RP}^2$




$$\mathbb{RP}^2 = \{ l : l \in \mathbb{R}^3,\ 0 \in l \}$$

point in $\mathbb{RP}^2$ = line in $\mathbb{R}^3$ passing through origin

point B — line $l_B$

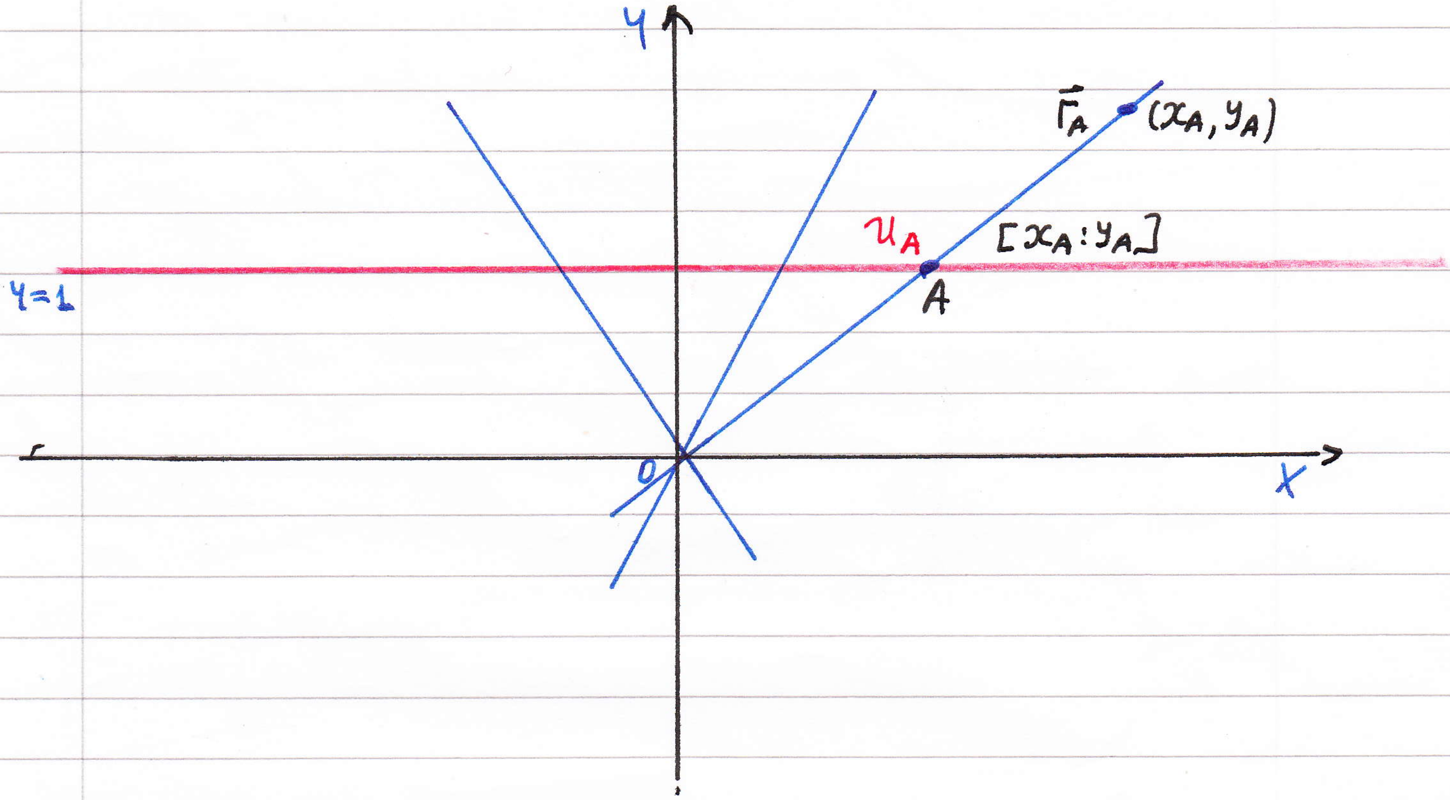
point A — line $l_A$

point C — line $l_C$

point at infinity — line $l$ which is parallel to the plane $\pi$ (z=1)

# Lecture C IX

Recall projective line $\mathbb{R}P^1$ in a bit more detail.




a point $[x_A : y_A]$ on $\mathbb{R}P^1$
$([x_A : y_A] = [\lambda x_A : \lambda y_A])$

line which passes through $\vec{0}$ and a point $\vec{r}_A = (x_A, y_A)$ in $\mathbb{R}^2$

$[x : y]$ — homogeneous coordinates of points in $\mathbb{R}P^1$

$$l : \begin{cases} x = t x_A \\ y = t y_A \end{cases}, \quad -\infty < t < \infty$$

$$\vec{l} = t \vec{r}_A$$

affine coordinate $u$

$$u_A = \frac{x_A}{y_A} \quad (y_A \neq 0)$$

line $l$ intersects the line $y = 1$

$y = t y_A = 1$, $t = \frac{1}{y_A}$. $(y_A \neq 0)$.

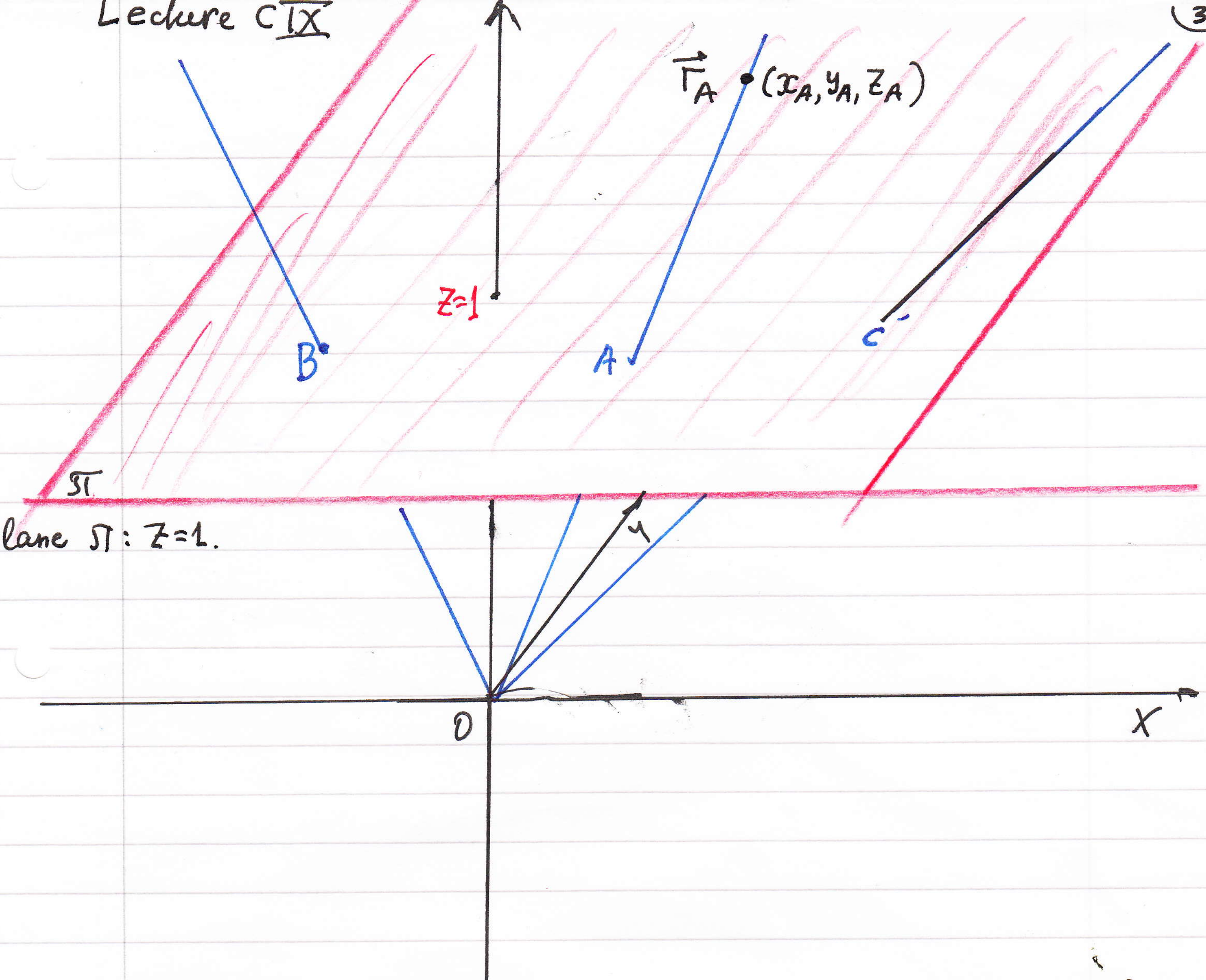
Point at infinity

$u_A = \infty$

line $l$: $y_A = 0$.

$l$ is parallel to the line $y = 1$

$$\mathbb{R}P^1 = \underbrace{\mathbb{R}}_{\text{lines } l \text{ which intersect } y=1} \cup \underbrace{\{\infty\}}_{\text{the line } l \parallel y=1}$$




plane $\pi$ : $z=1$.

a point $A = [x_A : y_A : z_A]$ in $\mathbb{R}P^2$

$([x_A : y_A : z_A] = [\lambda x_A : \lambda y_A : \lambda z_A]$

$[x:y:z]$ – homogeneous coordinates of points in $\mathbb{R}P^2$

affine coordinates $u, v$

$u_A = \frac{x_A}{z_A}$, $v_A = \frac{y_A}{z_A}$

(affine points)

Points at infinity

line $l$ which passes through $\vec{O}$ (origin) and a point $\vec{r}_A = (x_A, y_A, z_A)$

$$l : \begin{cases} x = t\, x_A \\ y = t\, y_A \\ z = t\, z_A \end{cases}, \quad -\infty < t < \infty$$

$$\vec{l} = t\, \vec{r}_A$$

line $l$ intersects the plane $\pi$ : $z=1$

$z = t z_A = 1$, $t = 1/z_A$ $\quad (z_A \neq 0)$

line $l$ : $z_A = 0$, $l \parallel \pi$

All the lines in the plane $z_A = 0$, passing through origin

# Lecture CIX

$\mathbb{RP}^2 = \{$lines in $\mathbb{R}^3$ passing through origin$\} =$

$= \mathbb{R}^2 \cup \mathbb{RP}^1$

$\mathbb{R}^2$: lines which intersect the plane $\pi: z=1$
finite points
affine points

$\mathbb{RP}^1$: lines which are parallel to $\pi$
points at infinity

$[x_A : y_A : z_A]$

$z_A \neq 0$
affine point
affine coordinates
$(u_A, v_A)$ : $u_A = \frac{x_A}{z_A}, v_A = \frac{y_A}{z_A}$

$z_A = 0$
point at infinity.

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If $A = [x_A : y_A : z_A]$ is a finite point (affine point) $(z_A \neq 0)$, then

$(u_A, v_A) = \left(\frac{x_A}{z_A}, \frac{y_A}{z_A}\right)$ is a point at the plane

$\pi: z = 1$, where the line $\ell : \begin{cases} x = t\,x_A \\ y = t\,y_A \\ z = t\,z_A \end{cases}$ intersects

$\vec{\ell} = t\,\vec{r}_A$

the plane $z = 1$.