Lecture C VI

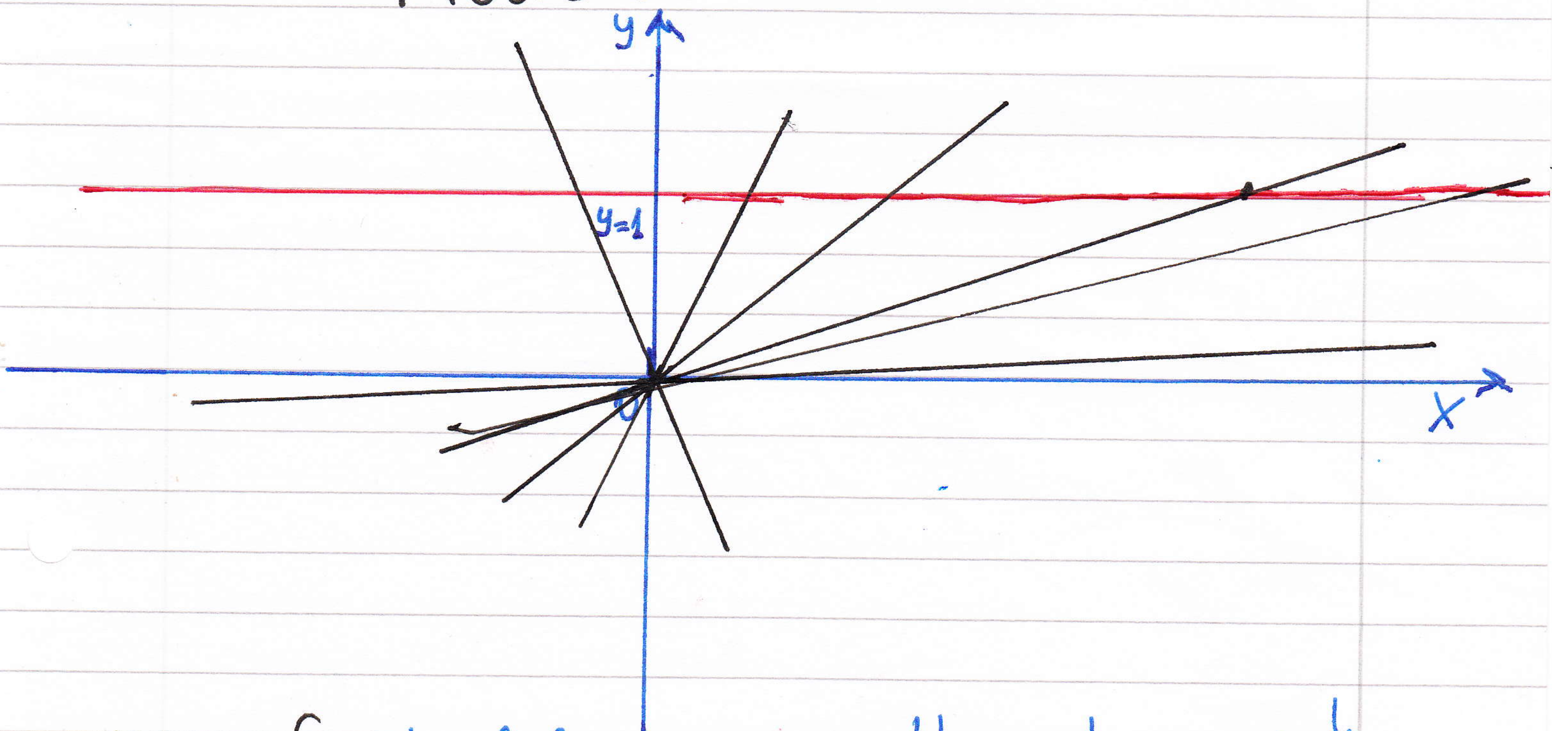
The PROJECTIVE LINE

$\mathbb{R}P^1$ — projected line

$$\mathbb{R}P^1 = \underbrace{\mathbb{R}}_{\text{usual line}} \cup \underbrace{\{\infty\}}_{\text{point at infinity}}$$

Projective line = Usual line completed by a point at infinity.

Model:



$\mathbb{R}P^1 = \{$set of lines passing through origin$\}$

$\mathbb{R}P^1 = \{\ell : 0 \in \ell\}$

Point of $\mathbb{R}P^1 \longleftrightarrow$ line ℓ: $0 \in \ell$

Every line $\underbrace{\ell : (0 \in \ell)}_{\text{point of } \mathbb{R}P^1}$, intersects ______

at the point u [except the line $\underline{\underline{\ell}}$ which goes along OX]

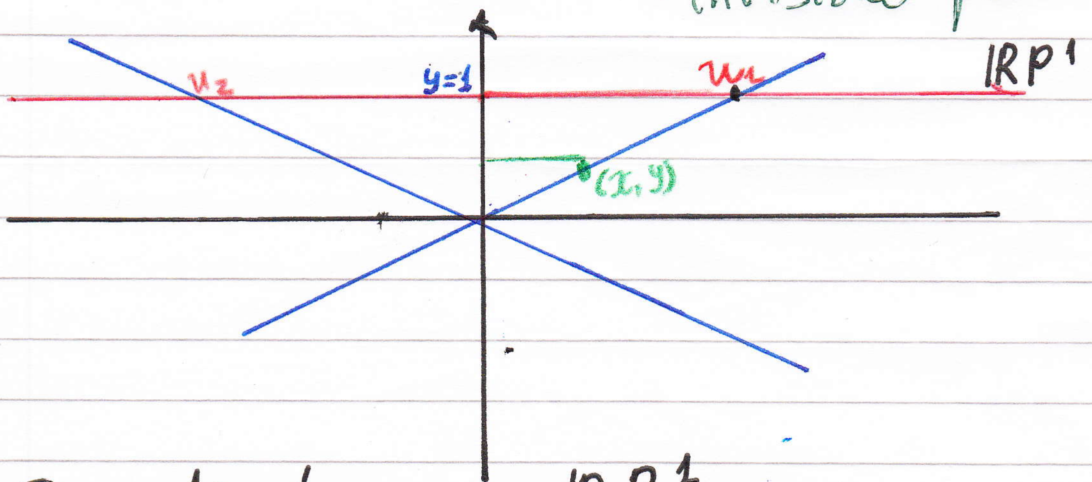
Lecture CVI

$$\left\{\begin{matrix}\text{set of lines} \\ \text{passing through} \\ \text{origin}\end{matrix}\right\} = \left\{\begin{matrix}\text{set of lines} \\ \text{passing through} \\ \text{origin which} \\ \text{intersect the} \\ \text{line } y=1\end{matrix}\right\} \cup \left\{\begin{matrix}\text{the line} \\ \text{which goes} \\ \text{along} \\ \text{OX axis}\end{matrix}\right\}$$

$$\mathbb{RP}^1 = \mathbb{R} \cup \{\infty\}$$

line l which intersects line $y=1$ represents point on the line $y=1$

line l which goes along OX axis represents point at infinity, 'invisible' point



Coordinates on $\mathbb{RP}^1$

$$\frac{x}{u} = \frac{y}{1} \qquad u = \frac{x}{y}$$

Take any point $(x,y) \in \mathbb{R}^2$. It defines a point $u = \frac{x}{y}$
$(x,y) \neq 0$

A point $(x,0)$ defines Infinity $\left(\frac{x}{0}\right)$

P (x,y) and $(\lambda x, \lambda y)$ define the same point

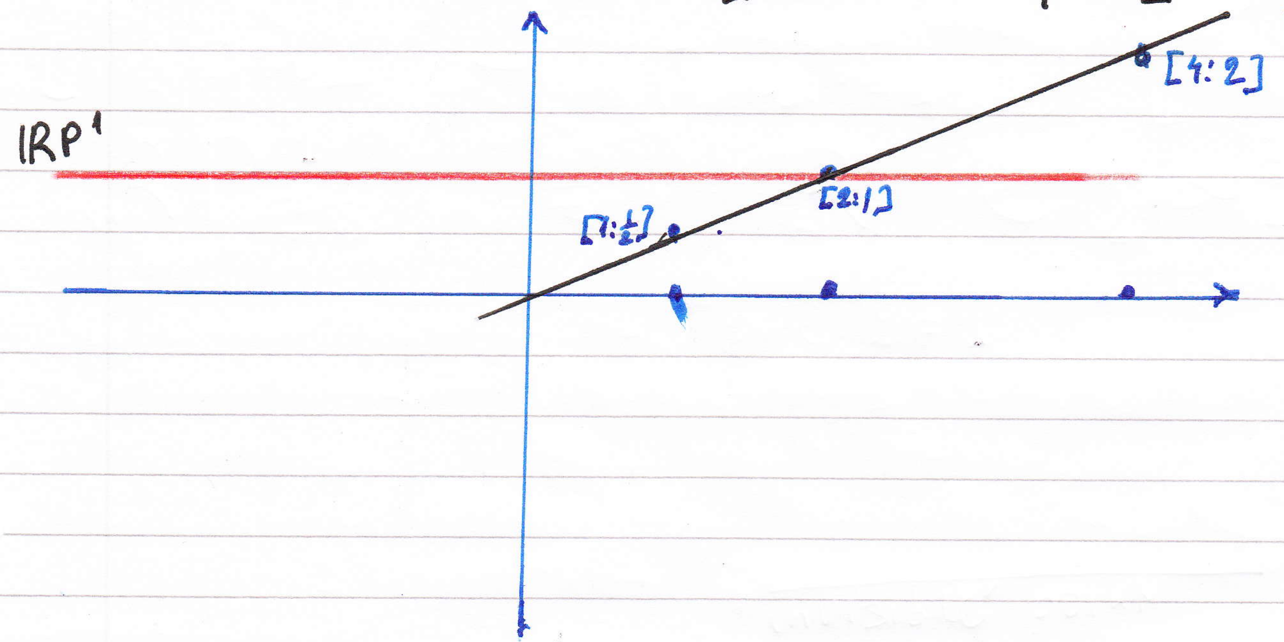
$$u = \frac{x}{y} = \frac{\lambda x}{\lambda y}$$

We denote this point

$[x : y]$ – Homogeneous coordinates

Examples

$[2:1] = [4:2] = [1:\tfrac{1}{2}]$ $\quad u = \frac{2}{1} = \frac{4}{2} = \frac{1}{\frac{1}{2}} = 2$



These are homogeneous coordinates of the same point on $\mathbb{R}P^1$ = Line ℓ $(0 \in \ell)$.

$[x:y]$ ———————— $u = \frac{x}{y}$

homogeneous coordinates — non-homogeneous coordinate

$[x:0]$ ———————— $u = \{\infty\}$