Homework 5b

`(the Homework 5 was divided on two homeworks: 5a, and 5b)`

1 Calculate differential forms $\omega = xdy - ydx$, $\sigma = xdx + ydy$ and vector fields $\mathbf{A} = x\partial_x + y\partial_y$, $\mathbf{B} = x\partial_y - y\partial_x$ in polar coordinates.

(this exercise was done during the XII-th lecture (see the subsection 2.3.5 "Differential forms in arbitrary coordinates" in Lecture notes)

2 Consider differential forms $\omega = xdy - ydx$, $\sigma = xdx + ydy$ and vector fields $\mathbf{A} = x\partial_x + y\partial_y$, $\mathbf{B} = x\partial_y - y\partial_x$.

Calculate $\omega(\mathbf{A}), \omega(\mathbf{B})$, $\sigma(\mathbf{A}), \sigma(\mathbf{B})$.

3 Consider a function $f = x^3 - y^3$.

Calculate the value of 1-form $\omega = df$ on the vector field $\mathbf{B} = x\partial_y - y\partial_x$.

4 Calculate the derivatives of the functions $f = x^2+y^2$, $g = y^2-x^2$ and $h = q\log|r| = q\log\left(\sqrt{x^2+y^2}\right)$ (q is a constant) along vector fields $\mathbf{A} = x\partial_x + y\partial_y$ and $\mathbf{B} = x\partial_y - y\partial_x$

a) calculating directional derivatives $\partial_{\mathbf{A}} f, \partial_{\mathbf{A}} g, \partial_{\mathbf{A}} h$, $\partial_{\mathbf{B}} f, \partial_{\mathbf{B}} g, \partial_{\mathbf{B}} h$,

b) calculating $df(\mathbf{A}), dg(\mathbf{A}), dh(\mathbf{A}), df(\mathbf{B}), dg(\mathbf{B}), dh(\mathbf{B})$.

5 Let f be a function on $\mathbf{E}^2$ given by $f(r,\varphi) = r^3\cos 3\varphi$, where r, φ are polar coordinates in $\mathbf{E}^2$.

Calculate the 1-form $\omega = df$.

Calculate the value of the 1-form $\omega = df$ on the vector field $\mathbf{X} = r\partial_r + \partial_\varphi$.

Express the 1-form ω in Cartesian coordinates x, y.

(You may use the fact that $\cos 3\varphi = 4\cos^3\varphi - 3\cos\varphi$.)

6 Show that 1-form $\omega = xdy + ydx$ is exact.

Show that 1-form $\omega = \sin y dx + x\cos y dy$ is exact.

Show that 1-form $\omega = x^3 dy$ is not an exact 1=form.

(We call 1-form ω exact if there exists a function F such that $\omega = dF$.)