

Homework 9

1 Let F be a projective transformation of \mathbf{RP} such that

$$[x' : y'] = F([x : y]) = [2x + 3y : 3x + 2y].$$

Let $P = [6 : 2]$ be a point on \mathbf{RP} .

Find the affine coordinate u_P of this point, and find the affine coordinate u'_P of the point $F(P)$.

Find a point A such that $A = F(\infty)$.

Find a point B such that $F(B) = \infty$.

2 Four points $A, B, C, D \in \mathbf{RP}^2$ are given in homogeneous coordinates by

$$A = [2 : -1 : 1], \quad B = [15 : -10 : 5], \quad C = \left[1 : -\frac{4}{5} : \frac{1}{5}\right], \quad D = [2 : 0 : 2].$$

Show that these points are collinear.

Calculate their cross-ratio.

3 Three points $A, B, C \in \mathbf{RP}^2$ are given in homogeneous coordinates by

$$A = [6 : 2 : 2], \quad B = [15 : 5 : 1], \quad C = [18 : 6 : 3].$$

Show that these points are collinear.

Find a point D on projective plane \mathbf{RP}^2 such that the point D is harmonic conjugate to the points A, B, C , i.e. the cross-ratio $(A, B, C, D) = -1$.

4 Let A, B, C, D be four collinear points on projective plane \mathbf{RP}^2 .

Let $(A, B, C, D) = \lambda$. Calculate (B, A, C, D) , (A, B, D, C) and (B, A, D, C) .

5 Two points A, B on the projective plane \mathbf{RP}^2 are given in homogeneous coordinates

$$A = [3 : 9 : 3], \quad B = [6 : 18 : 2].$$

a) Find the point $C = [x : y : z]$ on \mathbf{RP}^2 such that $x = 2z$ and the three points A, B, C are collinear.

b) on the projective line passing through the points A, B and C find a point D such that the cross-ratio (A, B, C, D) of these points is equal to $-\frac{1}{2}$.

c) Find cross ratio of the points (A, C, B, D) .