$$\Lambda^k_R \xrightarrow{d} \Lambda^{k+1} \xrightarrow{d} \Lambda^{k+2}$$

$$d^2 = 0$$

$$0 \to \underset{\substack{\text{functions} \\ f(x)}}{\Lambda^0(M)} \longrightarrow \underset{\substack{\text{1-forms} \\ r_i(x)dx^i}}{\Lambda^1(M)} \longrightarrow \underset{\substack{\text{2-forms} \\ \rho(x)dx\wedge dy}}{\Lambda^2(M)} \to 0$$

$$0 \to \boxed{\text{functions}} \xrightarrow{\text{grad}} \boxed{\text{Vector fields}} \xrightarrow{\text{div}} \boxed{\text{function}}$$

$A^i = g^{ik} r_k$

$a(x)$

$\partial_i A^i = a$

$\rho(x,y)dx\wedge dy = a(x) \underbrace{dx\wedge dy}_{\text{volume form}}$

in general

$$0 \longrightarrow \Lambda^0(M) \xrightarrow{d} \Lambda^1(M) \xrightarrow{d} \dots \xrightarrow{d} \Lambda^{n-1}(M) \xrightarrow{d} \Lambda^n(M) \to 0$$

$\Lambda^r(M)$ – r-forms

$$\Lambda^{k-1} \xrightarrow{d} \Lambda^{k} \xrightarrow{d} \Lambda^{k+1}$$

$$H^k = Z^k / B^k$$

$Z^k = \{\omega : \omega \in \Lambda^k, d\omega = 0\}$
(closed forms) (cocycle)

$B^k = \{\omega : \omega \in \Lambda^k, \exists \Gamma \in \Lambda^{k-1} : \omega = d\Gamma\}$
ω - exact form (coboundary)

$B^k = \operatorname{Im} d\big|_{\Lambda^{k-1}}, \; Z^k = \ker d\big|_{\Lambda^k}$

$H^k(M)$ - space of de Rham cohomology

$b_k = \dim H^k(M)$ - Betti number

	b_0	b_1	b_2	b_3	...
Circle S^1	1	1	0	0	...
Sphere S^2	1	0	1	0	
Torus $S^1 \times S^1$	1	2	1	0	...

Euler characteristic.

$$\chi(M) = \sum (-1)^k b_k = b_0 - b_1 + b_2 - b_3 \dots$$

$$0 \longrightarrow \Lambda^0(M) \to \Lambda^1(M) \to \dots \to \Lambda^{k-1}(M) \to \Lambda^k(M) \to 0$$

Suppose that $\dim \Lambda^i(M) < \infty$

$$\Lambda^{k-1}(M) \xrightarrow{d} \Lambda^k(M) \xrightarrow{d} \Lambda^{k+1}(M)$$

$$H^k = Z^k / B^k = \cancel{\dim} \frac{\ker d|_{\Lambda^k}}{\operatorname{Im} d|_{\Lambda^{k-1}}}$$

$$\Downarrow$$

$$\dim \Lambda^k = \dim \ker d|_{\Lambda^k} + \dim \operatorname{Im} d|_{\Lambda^k}$$

$$\dim \Lambda^k = \dim Z^k + \dim B^{k+1}$$

$$\Downarrow$$

$$\sum (-1)^i \dim \Lambda^i = \sum (-1)^i \dim Z^i + \sum (-1)^i \dim B^{i+1} =$$

$$= \sum (-1)^i \dim Z^i - \sum (-1)^i \dim B^i =$$

$$= \sum (-1)^i [\dim Z^i - \dim B^i] =$$

$$= \sum (-1)^i b_i = \chi(M)$$