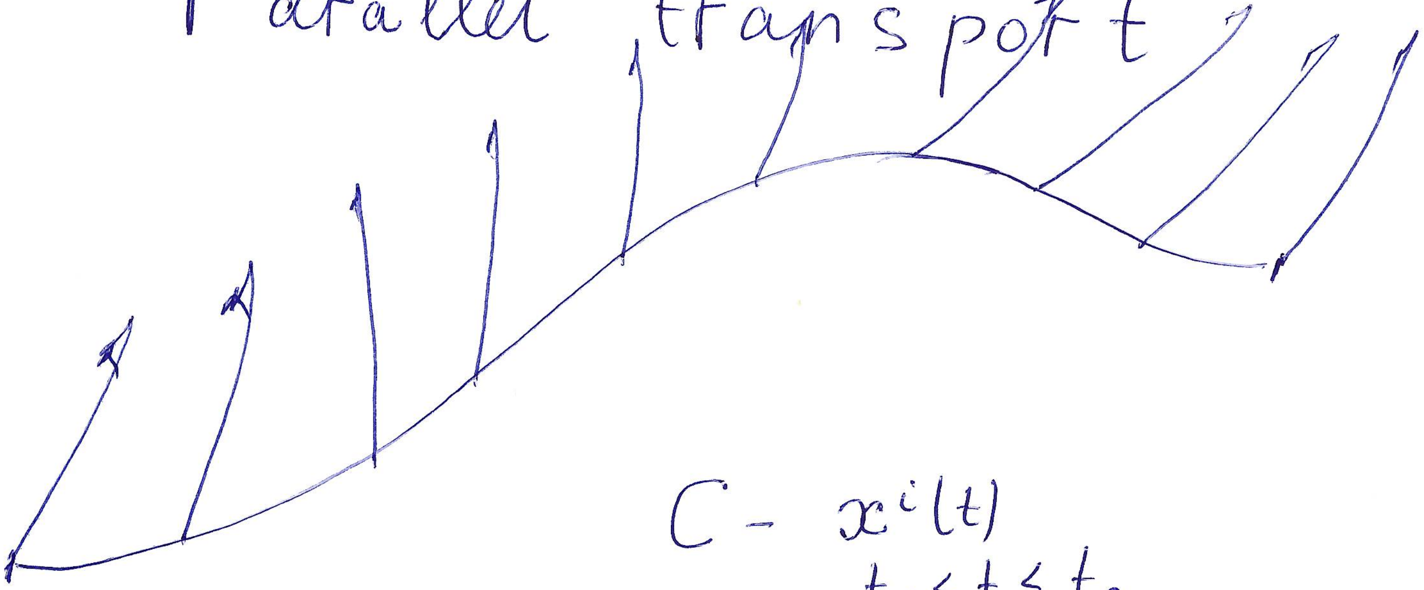
Parallel transport



$C - x^i(t)$

$t_1 \leq t \leq t_2$

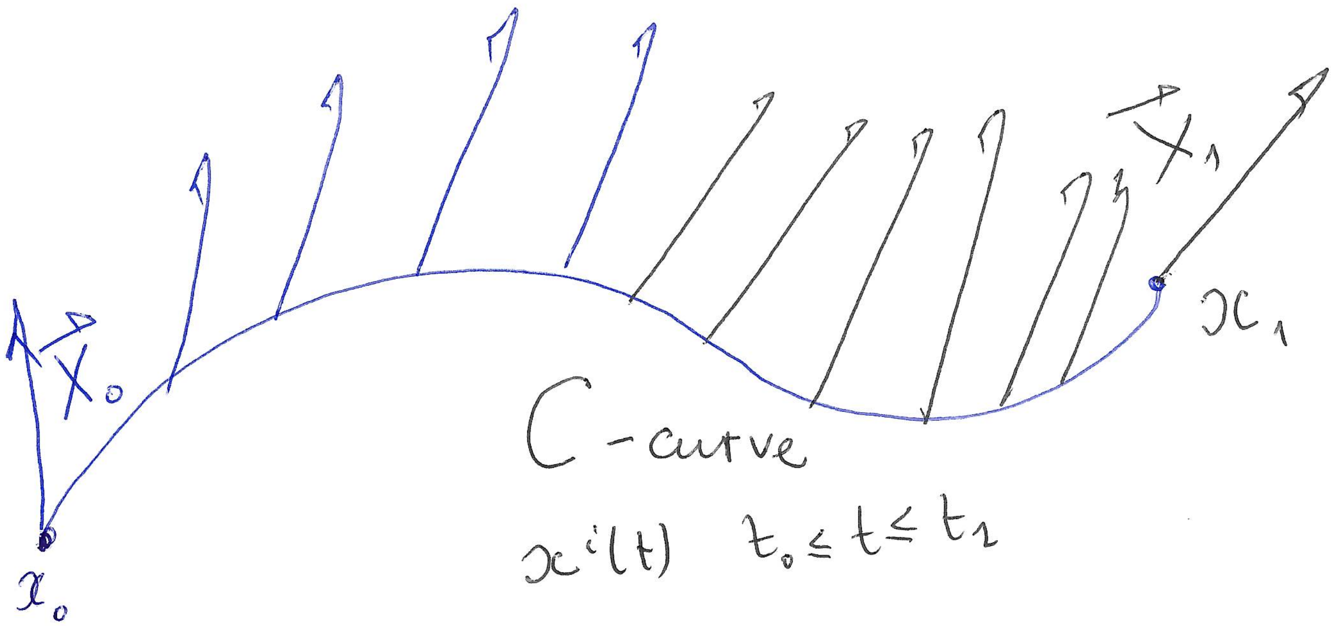
$\vec{X}(t)$ is attached at the point $x^i(t)$

$$\frac{\nabla \vec{X}(t)}{dt} = \nabla_{\vec{v}} \vec{X}(t) = 0$$

$$\vec{X}(t) = X^m(t) \frac{\partial}{\partial x^m}\Big|_{x(t)}, \quad \vec{v} = \frac{dx}{dt} = \frac{dx^i}{dt} \partial_i$$

$$\frac{dX^i(t)}{dt} + v^k(t)\, \Gamma^i_{km}(x(t))\, X^m(t) \equiv 0$$

We say that $\vec{X}(t)$ is covariantly constant along the curve C.



$$T_{x_0}M \ni \vec{X}_0 \dashrightarrow \vec{X}_1 \in T_{x_1}M$$

$$P_C : T_{x_0}M \longrightarrow T_{x_1}M$$

Linear operator of parallel transport

If ∇ is Levi-Civita connection then parallel transport preserves scalar product

$$\langle \vec{X}_0, \vec{Y}_0 \rangle \Big|_{x_0} = \langle \vec{X}_1, \vec{Y}_1 \rangle \Big|_{x_1}$$

Geodesic is generalisation of straight line.

straight line — shortest
— straightest
— trajectory of free particle

shortest .... $x_0^i(t)$ geodesics if $\int \sqrt{g_{ik}\,\dot{x}_0^i(t)\,\dot{x}_0^k(t)} \le \int \sqrt{g_{ik}\,\dot{x}^i(t)\,\dot{x}^k(t)}\,dt$

straightest $x_0^i(t)$ geodesics if $\nabla_V V = \frac{\nabla V}{dt} = \frac{dV^i}{dt} + V^k(t)\Gamma^i_{km}V^m(t) = 0$

trajectory of free particle ... particle moves along geodesics

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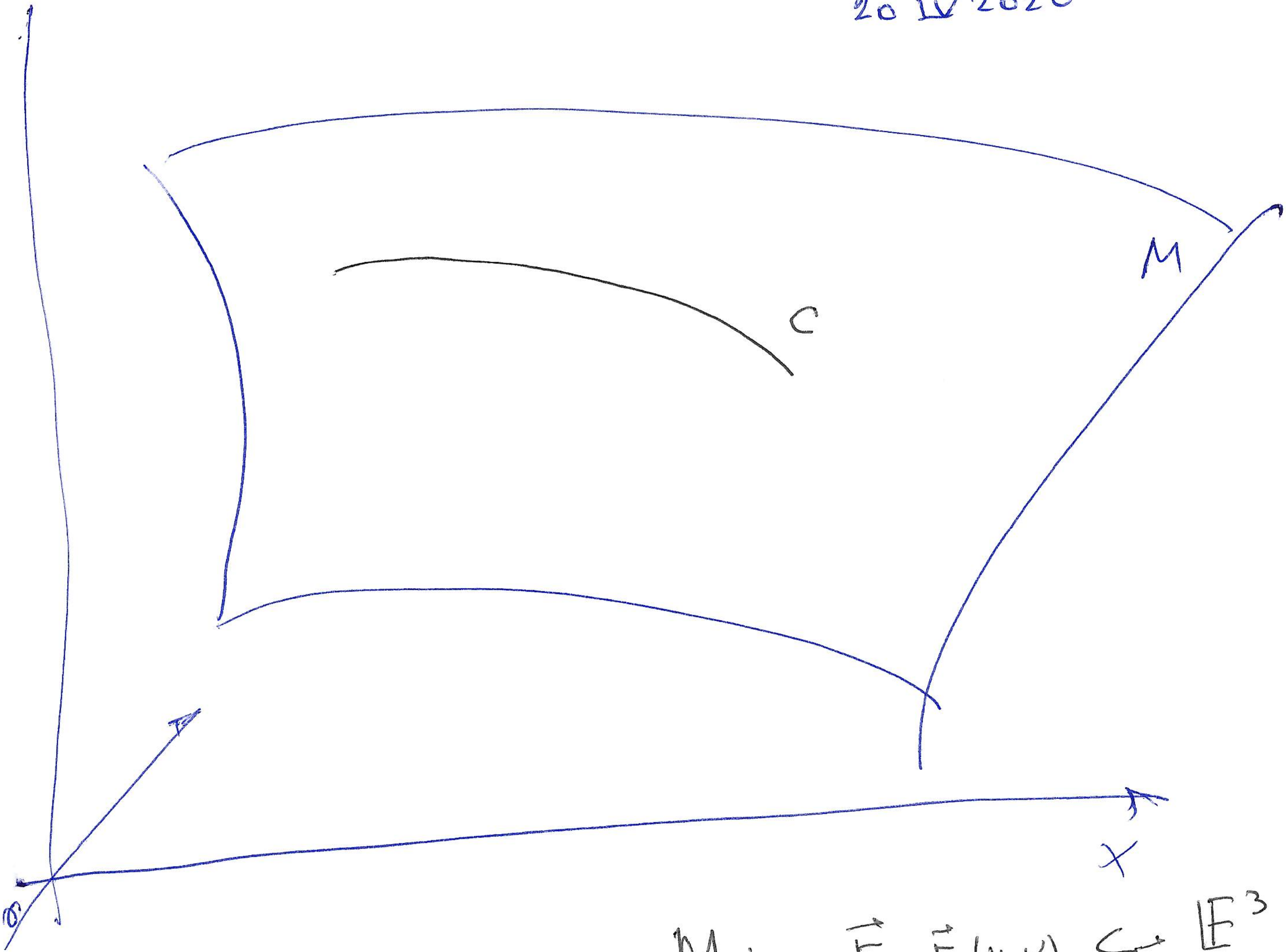


$$M: \quad \vec{r} = \vec{r}(u,v) \hookrightarrow \mathbb{E}^3$$

$$C - \vec{r} = \vec{r}(u(t), v(t))$$ - geodesics

$$\nabla_{\vec{v}} \vec{v} = 0 = \nabla^M_{\vec{v}} \vec{v} = \left(\nabla^{\text{can. fl.}}_{\vec{v}} \vec{v} \right)_{\text{tang.}} = 0$$

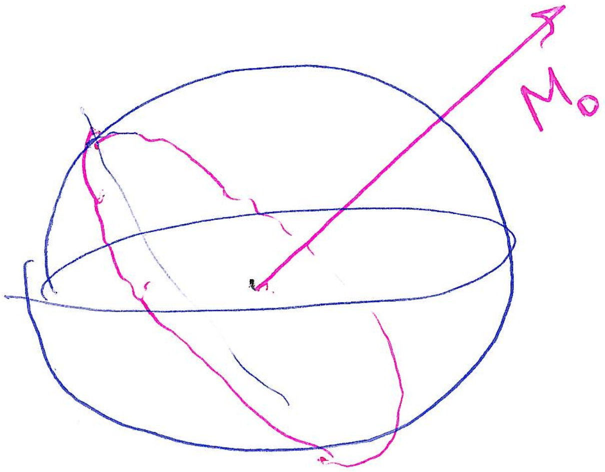
$\vec{r}(t)$ geodesics

$\Updownarrow$

$\frac{d^2\vec{r}(t)}{dt^2} = \vec{a}(t)$ is orthogonal to surface.

(Force is orthogonal)

Geodesics on sphere — great circles.



$$\vec M(t) = \vec r(t) \times \vec v(t)$$

$$\frac{d\vec M(t)}{dt} = \frac{d\vec r(t)}{dt} \times \vec v(t) + \vec r(t) \times \frac{d^2 \vec r(t)}{dt^2} =$$

$$= \underbrace{\vec v(t) \times \vec v(t)}_{0} + \vec r(t) \times \underbrace{\vec a(t)}_{\vec a(t) \sim \vec r(t)} = 0$$

$$\Downarrow$$

$\vec M_0 = \vec M(t)$ is constant vector

$$\Downarrow$$

$$\vec r(t) \perp \vec M_0$$